WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2021-22

PHSACOR05T-PHYSICS (CC5)

MATHEMATICAL PHYSICS-II

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
Answers must be precise and to the point to earn credit.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) How will you change the function $f(x)$ in the interval $(-\pi,\pi)$ to $(-l,l)$ in Fourier series?

(b) Sketch the periodic extension of $f(t)=0$ for $t<0$, $f(t)=1$ for $t>0$, if the fundamental interval is $(-1, 1)$.

(c) Let $F(x,y,y')=2x+xy'+(y')^2+y$, then obtain the corresponding Euler Lagrange equations that follow from Hamilton's principle.

(d) Prove that if the Lagrangian of a conservative system does not contain time explicitly, the total energy is conserved.

(e) Prove that $P_{2m+1}(0)=0$.

(f) Find the nature and location of the singularities of the following differential equation $(1-x^2)y''-2xy'+l(l+1)y=0$.

(g) For integer n Bessel function of first kind are given as,

$$J_n(x)=\sum_{j=0}^{\infty}\frac{(-1)^j (x/2)^{2j+n}}{j!\Gamma(n+j+1)}$$

Show that $J_{-n}(x)=(-1)^n J_n(x)$.

(h) State Hamilton's principle.

(i) What do you understand by degrees of freedom of a dynamical system?

(j) Determine the Wronskian for the differential equation $y''+y=0$.

(k) If $H_n(x)$ are Hermite polynomials then evaluate $\int_{-\infty}^{\infty} H_2(x)H_3(x)e^{-hx^2}dx$.

(l) Using Generating function of $H_n(x)$ i.e. $e^{-t^2+2tx}=\sum_{n=0}^{\infty} H_n \frac{t^n}{n!}$,

show that $H_n(-x)=(-1)^n H_n(x)$

(m) With the help of Legendre transformation, determine the Hamiltonian corresponding to the Lagrangian $L=\frac{1}{m}\dot{x}^2$, where m is a constant.

(n) Consider $y=\sum_{r=0}^{\infty} a_k x^{k+m}$. Obtain the indicial equation and it's roots for the Legendre's differential equation.

2. (a) Write the differential equation obeyed by Legendre polynomials. Show that 3

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

(b) If $f(x)=0$ for $0<x<\pi/2$ 4

$=\pi-x$ for $\pi/2<x<\pi$,

Then show that $f(x)=\frac{\pi}{4}-\frac{2}{\pi}\left(\frac{1}{1^2}\cos 2x+\frac{1}{3^2}\cos 6x+\ldots\right)$

(c) Solve the differential equation 3

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \text{ and obtain } \phi(x,y)$$

Given $\phi(x,y)=\phi(0,y)=0$

3. (a) Using the generating function for the Legendre polynomials $P_n(x)$ to show that 2+2

Prove that $\int_{-1}^{1}[P_n(x)]^2 dx=\frac{2}{2n+1}$

and hence show that $\int_{-1}^{1} P_3^2(x)\,dx=\frac{2}{7}$

(b) Show that the Bessel function 3

$$J_n(x)=\sum_{j=0}^{\infty}\frac{(-1)^j (x/2)^{2j+n}}{j!\,\Gamma(n+j+1)}$$

satisfy the Bessel's equation $x^2y''+xy'+(x^2-n^2)y=0$.

(c) Apply Legendre Transformation on the Internal energy function U(S,V) to obtain Gibbs free energy G(T,P). 3

4. (a) Consider the differential equation $2x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-(x+1)y=0$, obtain a series solution in powers of x using Frobenius method. 5

(b) Derive Hamilton's canonical equations of motion. Obtain Hamilton's equation for a particle in a central force field. 2+3

5. (a) A charge $+q$ is distributed uniformly along the z-axis from $z=-a$ to $z=+a$. Show that the electric potential at a point $\vec{r}$ is given by 4

$$V(r,\theta)=\frac{q}{4\pi\varepsilon_0 r}\left[1+\frac{1}{3}\left(\frac{a}{r}\right)^2 P_2(\cos\theta)+\cdots\ldots\right].,$$

where $P_n(\cos\theta)$ are Legendre polynomial.

(b) Using Hamilton's canonical equations of motion, show that the Hamiltonian $(H(x,p,t))$ is conserved provided $\frac{\partial H}{\partial t}=0$. 3

(c) If $u=f(r)$ and $x=r\cos\theta,\ y=r\sin\theta$, prove that 3

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r).$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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